

## Gomory's cutting plane algorithm

Given the following Integer Programs:

1. Evaluate the linear relaxation  $\mathbf{x}_{LP}$
2. Generate a Gomory cut for each fractional component of  $\mathbf{x}_{LP}$

$$\begin{aligned} \max \quad & 15x_1 + 35x_2 \\ & 2x_1 + 2x_2 + x_3 = 1 \\ & 4x_1 + 4x_2 + x_4 = 1 \\ & 5x_2 + x_5 = 1 \\ & x_i \in \mathbb{Z}_+ \quad \forall i = 1, \dots, 5 \end{aligned}$$

$$\begin{aligned} \max \quad & x_1 + \frac{3}{2}x_2 \\ & 2x_1 + \frac{1}{2}x_2 \leq \frac{1}{2} \\ & 2x_1 + 3x_2 \leq 2 \\ & x \in \mathbb{Z}_+^2 \end{aligned}$$

$$\begin{aligned} \max \quad & 2x_1 + x_2 \\ & x_1 + x_2 \leq 5 \\ & -x_1 + x_2 \leq 0 \\ & 6x_1 + 2x_2 \leq 21 \\ & x \in \mathbb{Z}_+^2 \end{aligned}$$

$$\begin{aligned} \max \quad & 2x_1 + 3x_2 \\ & 4x_1 + 3x_2 \leq 12 \\ & 2x_1 + 3x_2 \leq 8 \\ & x \in \mathbb{Z}_+^2 \end{aligned}$$

## Preprocessing

Preprocess the following linear programs:

$$\max 8x_1 + x_2 - 6x_3$$

s.t.

$$10x_1 - 6x_2 + x_3 \leq -11$$

$$4x_1 + x_2 - 5x_3 \leq -6$$

$$6x_1 - 3x_2 + 4x_3 \geq -7$$

$$0 \leq x_1 \leq 5$$

$$0 \leq x_2 \leq 3$$

$$\frac{3}{2} \leq x_3$$

$$\max 7x_1 + x_2 - 5x_3$$

s.t.

$$-8x_1 + 8x_2 - x_3 \geq 13$$

$$-3x_1 - x_2 + 5x_3 \geq 8$$

$$-6x_1 + 3x_2 - 3x_3 \leq 9$$

$$0 \leq x_1 \leq 5$$

$$0 \leq x_2 \leq 2$$

$$2 \leq x_3$$

$$\min -6x_1 - x_2 + 4x_3$$

s.t.

$$-9x_1 + 7x_2 - x_3 \geq 12$$

$$-3x_1 - x_2 + 4x_3 \geq 7$$

$$-5x_1 + 4x_2 - 3x_3 \leq 8$$

$$0 \leq x_1 \leq 4$$

$$0 \leq x_2 \leq \frac{5}{2}$$

$$2 \leq x_3$$

## Branch-and-bound

Given the following binary programs:

1. Derive (if possible) inequalities from logical implications
2. Solve with the branch-and-bound algorithm

$$\max 21x_1 + 17x_2 + 15x_3 + 25x_4$$

s.t.

$$5x_1 - 8x_2 + 14x_3 \leq 15$$

$$-13x_2 - 6x_3 - 7x_4 \leq 13$$

$$x_1 + x_4 \leq 1$$

$$x_2 + x_4 \leq 1$$

$$x_1 + x_2 \leq 1$$

$$x \in \{0,1\}^4$$

$$\max 13x_1 + 16x_2 + 25x_3 + 35x_4$$

s.t.

$$6x_1 - 4x_2 + 12x_3 \leq 13$$

$$10x_2 + 6x_3 - 5x_4 \leq 16$$

$$x_1 + x_4 \leq 1$$

$$x_2 + x_4 \leq 1$$

$$x_1 + x_2 \leq 1$$

$$x \in \{0,1\}^4$$

$$\max 15x_1 + 15x_2 + 15x_3 + 16x_4$$

s.t.

$$4x_1 + 9x_2 + 8x_3 \geq 5$$

$$-14x_2 - 8x_3 - 9x_4 \leq 14$$

$$x_1 + x_2 \leq 1$$

$$x_2 + x_3 \leq 1$$

$$x_1 + x_3 \leq 1$$

$$x \in \{0,1\}^4$$

## Cover inequalities

Given the following knapsack problems, derive (if possible) cover inequalities violated by the optimal solution of the linear relaxation.

$$\begin{aligned} \max & 16x_1 + 23x_2 + 19x_3 + 18x_4 + 21x_5 + 28x_6 + 13x_7 \\ & 18x_1 + 22x_2 + 14x_3 + 17x_4 + 18x_5 + 29x_6 + 16x_7 \leq 68 \\ & x \in \{0,1\}^7 \end{aligned}$$

$$\begin{aligned} \max & 10x_1 + 14x_2 + 18x_3 + 9x_4 + 13x_5 + 21x_6 + 17x_7 \\ \text{s.t.} & \\ & 12x_1 + 11x_2 + 9x_3 + 14x_4 + 18x_5 + 17x_6 + 20x_7 \leq 36 \\ & x \in \{0,1\}^7 \end{aligned}$$

$$\begin{aligned} \max & 12x_1 + 35x_2 + 13x_3 + 17x_4 + 15x_5 + 26x_6 + 15x_7 \\ & 16x_1 + 23x_2 + 13x_3 + 15x_4 + 12x_5 + 32x_6 + 15x_7 \leq 67 \\ & x \in \{0,1\}^7 \end{aligned}$$

$$\begin{aligned} \max & 14x_1 + 25x_2 + 23x_3 + 16x_4 + 25x_5 + 29x_6 + 12x_7 \\ & 17x_1 + 24x_2 + 15x_3 + 15x_4 + 14x_5 + 31x_6 + 15x_7 \leq 72 \\ & x \in \{0,1\}^7 \end{aligned}$$